

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

2213454579

FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

1	(a)	Sketch the curve with equation	<i>y</i> =	$\frac{x+1}{x-1}$.
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[2]

(b) Sketch the curve with equation $y = \frac{|x|+1}{|x|-1}$ and find the set of values of x for which $\frac{|x|+1}{|x|-1} < -2$.

	cubic equation $x^3 + 5x^2 + 10x - 2 = 0$ has roots α , β , γ .							
(a)	Find the value of $\alpha^2 + \beta^2 + \gamma^2$.	[
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Snow	that the mati	$ \begin{array}{ccc} \operatorname{rix} \left(\begin{array}{ccc} \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{array} \right) $	is singular.		
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3	A cı	urve C has equation $y = \frac{ax^2 + x - 1}{x - 1}$, where a is a positive constant.	
	(a)	Find the equations of the asymptotes of C .	[3]
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	(b)	Show that there is no point on C for which $1 < y < 1 + 4a$.	[4]
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(c)	Sketch <i>C</i> . You do not need to find the coordinates of the intersections with the axes.	[3]

()	Using the method of differences, or otherwise, find $\sum_{r=1}^{n} u_r$ in terms of n and x .	
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<i>(</i> 1.)		•••
(b)	Deduce the set of non-zero values of x for which the infinite series	•••
(b)	$u_1 + u_2 + u_3 + \dots$	•••
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(c)	Using a standard result from the list of formulae (MF19), find $\sum_{r=1}^{n} \ln u_r$ in terms	of n and x .	[3]
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(a)	State the type of the geometrical transformation in the x - y plane represented by \mathbf{A} .	
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(b)	Prove by mathematical induction that, for all positive integers n ,	
	$\mathbf{A}^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}.$	
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Let $\mathbf{B} = \begin{pmatrix} b & b \\ a^{-1} & a^{-1} \end{pmatrix}$, where *b* is a positive constant. (c) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by $\mathbf{A}^{n}\mathbf{B}$.

6	The	curve C has Cartesian equation $x^2 + xy + y^2 = a$, where a is a positive constant.								
	(a)	Show that the polar equation of <i>C</i> is $r^2 = \frac{2a}{2 + \sin 2\theta}$. [3]								
	(b)	Sketch the part of C for $0 \le \theta \le \frac{1}{4}\pi$. [2]								

The region *R* is enclosed by this part of *C*, the initial line and the half-line $\theta = \frac{1}{4}\pi$.

(c) It is given that $\sin 2\theta$ may be expressed as $\frac{2 \tan \theta}{1 + \tan^2 \theta}$. Use this result to show that the area of *R* is

$$\frac{1}{2}a\int_0^{\frac{1}{4}\pi} \frac{1+\tan^2\theta}{1+\tan\theta+\tan^2\theta} d\theta$$

and use the substitution $t = \tan \theta$ to find the exact value of this area.	[8]
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resp	ectively.										
(a)	Given that $\lambda^2 - 5\lambda +$	at the shortes $-4 = 0$.	t distance	between	the line A	B and	the line	CD	is 3,	show	that [7]
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15 Let Π_1 be the plane ABD when $\lambda = 1$. Let Π_2 be the plane *ABD* when $\lambda = 4$. (b) (i) Write down an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [2] (ii) Find an equation of Π_2 , giving your answer in the form ax + by + cz = d. [4]

Additional page

If you use the following page to complete the answer to an shown.	y question, the question number must be clearly

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